

# Stress Analysis of Radial and Non-Radial Nozzle Connections in Ellipsoidal Head Pressure Vessel

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## ABSTRACT

*One of the critical aspects in designing pressure vessels is the nozzle connections in the ellipsoidal head of two intersecting shells. This paper presents a parametric study using Finite Element Analysis (FEA) method to determine the influence of the non-geometric parameters for a radial and non-radial nozzle connections in ellipsoidal heads vessel subjected to internal pressure and various external loadings. All the results analysis for both radial and non-radial nozzle for each load case applied, presented as graphs of non-dimensional parameters against stress concentration factor (SCF). In order to validate the FEA method, an analytical investigation of thin shell theory used for radial nozzle subjected to internal pressure loading. The comparison results between both methods show that the FEA method was reliable and valid in this research with only less than 10% differences.*

**Keywords:** *Finite Element Analysis (FEA), Ellipsoidal head, Radial and Non-Radial nozzle connections, Non-dimensional geometric parameters, Analytical calculation.*

## Nomenclature

$r_c$	: Mean radius of nozzle
$b$	: Depth of ellipsoidal head
$a$	: Mean radius of ellipsoidal head
$h_c$	: Thickness of nozzle
$h_e$	: Thickness of ellipsoidal head
$p$	: Uniform internal pressure
$E$	: Young's Modulus

$\nu$	: Poison's ratio
$N_{1\varphi e}, N_{2\theta e}$	: Forces in meridional & circumferential of ellipsoidal head
$N_{1xc}, N_{2\theta c}$	: Forces in longitudinal & circumferential of nozzle
$q$	: Ratio of mean radius of nozzle per mean radius of ellipsoidal head
$R_1$	: Principle of radius curvature in the meridional direction ellipsoidal head
$R_2$	: Principle of radius curvature in the circumferential direction ellipsoidal head
$\varphi_e$	: Angle at edge joint in ellipsoidal head
$\varphi$	: Angle at any location in ellipsoidal head
$\alpha, D_e$	: Coefficient of ellipsoidal head
$\beta, D_c$	: Coefficient of nozzle
$Q_0$	: Total edge force
$M_0$	: Total bending moment
$N_1, N_2$	: Corresponding force at membrane in the meridional & circumferential direction ellipsoidal head
$M_1, M_2$	: Corresponding moment at membrane in the meridional & circumferential direction ellipsoidal head
$\sigma_1, \sigma_2$	: Corresponding principle stress in the meridional & circumferential direction ellipsoidal head
$V$	: Equivalent (Von Mises) Stress

## **Introduction**

The stress imposed by related piping systems is vital to complete the design evaluation. To ensure that the design is adequate for the proposed services, ASME Code Section VIII Division 1 and 2, provide a guideline for designing these pressure vessels [1, 2]. Additionally, another important design guide ever published for designing the pressure vessel are The Welding Research Council (WRC) Bulletin No.107 [3] and Bulletin No. 297 [4]. It had become indispensable and reliable tools in these recent years.

A local stress state characterized by high stress concentrations occurs at the intersection region of the nozzle connection because of different load applied to the structures. The determination of main vessel-nozzle dimensions at the designing phase is the influence of the inner pressure as the main loading. Nonetheless, the piping system to the nozzle should be considered the impact of external forces and moments "in addition to the stress caused" by internal pressure. There is no method recommended by the Bulletin in analysing an actual nozzle connection. In addition, WRC-107 and WRC-297 only provides stress concentration and the procedure for

cylindrical and spherical vessels and not for ellipsoidal head. As a consequence, the designers have to make their own judgment.

Many published works presented analytical, numerical and experiment in recent years. However, most of these investigations related to the internal pressure and radial nozzle connections due to the cylindrical shell, spherical and torispherical shell only. In designing the pressure vessel equipment, the nozzle connection requires a careful study of many regions and thus, a parametric study was carried out in this research. The objective is to determine the influence of the non-geometric parameters for the radial and non-radial nozzle connections in ellipsoidal head vessel due to the internal pressure and various external loadings using the Finite Element Analysis (FEA) method. For the purpose of validation due to the FEA method, an analytical investigation using thin shell theory applied in this research for the radial nozzle due to internal pressure loading.

## **Finite Element Approach**

### **Geometry and 3D model**

To construct the model for analysis, the radial and non-radial nozzle connections in ellipsoidal heads vessel (refer to: Fig. 1(a) and Fig. 1(b)) created in 3-D solid models using finite element software-ANSYS V14. Both models generated with full scale based on the actual dimension and with a complete structural component of the pressure vessel such as top and bottom ellipsoidal head, nozzle, shell, flange and skirt (refer to: Fig. 1(c)).



Figure 1(a) Radial Nozzle



Figure 1(b) Non-Radial Nozzle

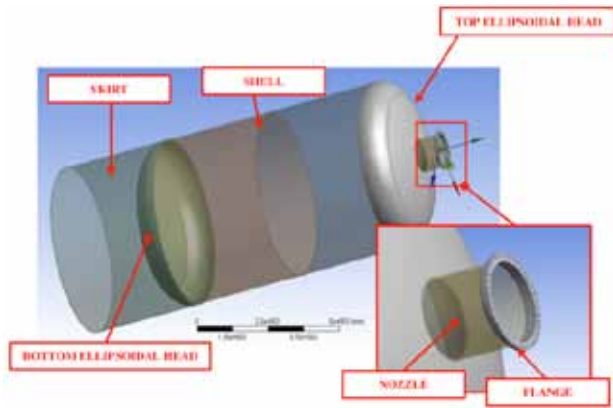


Figure 1(c) Design Modeler to generate models

### Meshing

In order to produce more accurate results, the meshing for radial and non-radial models discretized using higher-order tetrahedral and hexahedral solid elements (refer to: Fig. 2). The mesh graded, with fine elements in the structure component of flange, nozzle and top & bottom ellipsoidal head. Overall, the finite element mesh model for radial nozzle consists of 422505 nodes and 156596 elements. While for non-radial nozzle, consists of 425452 nodes and 152408 elements.

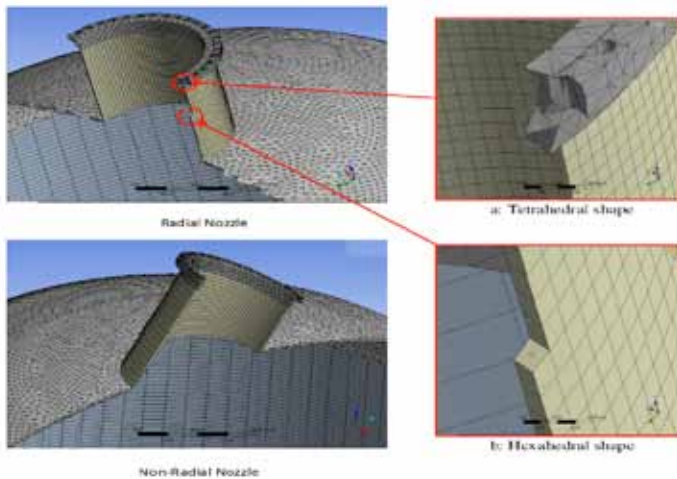


Figure 2 Element shapes to build meshing

**Material Data**

The materials of the structure component and mechanical properties used for this analysis given in Table 1 and Table 2.

Table 1 Materials of structure component

No.	Component	Material
1.	Top Ellipsoidal Head	SA 516 GR 70
2.	Bottom Ellipsoidal Head	
3.	Shell	
4.	Skirt	
5.	Nozzle	
6.	Flange	SA 105

Table 2 Materials properties

Mechanical Properties of Materials	SA 516 GR 70	SA 105
Density, $\rho$ [ kg/m <sup>3</sup> ]	8030	8030
Poison's ratio, $\nu$	0.30	0.30
Elastic Modulus, E [ Gpa ]	210	179.2
Tensile Ultimate Strength [ Mpa ]	485	485
Tensile Yield Strength [ Mpa ]	260	250

**Boundary Condition**

Selecting the appropriate boundary condition on the model is critical in FE analysis (refer to: Fig. 3). The boundary condition for both radial and non-radial models set “on the bottom surface of the skirt” to be constrained in x, y and z direction (refer to: Fig. 3).

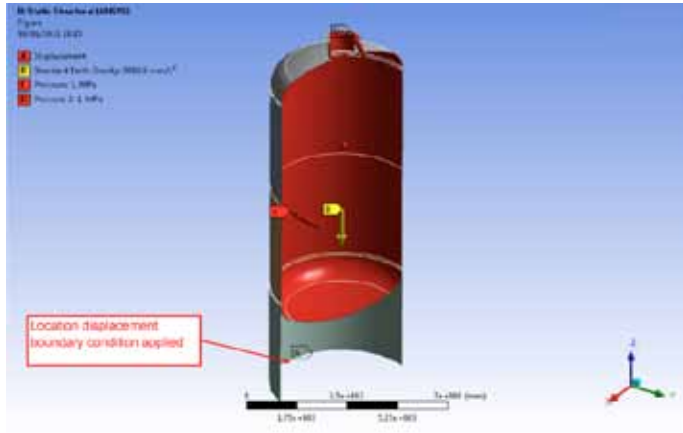


Figure 3 Location displacement boundary condition applied

### Load Cases

There are five load cases to be analyzed as per Figure 4. The set-up load for radial and non-radial nozzle in FE shown as per Figure 5(a) -(e).

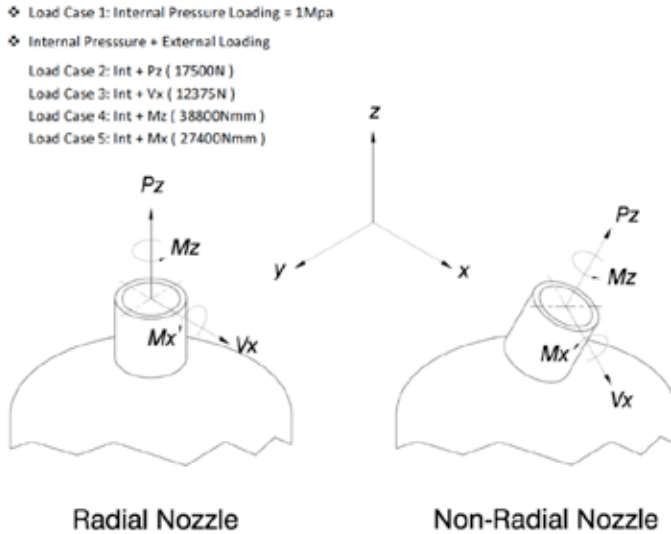
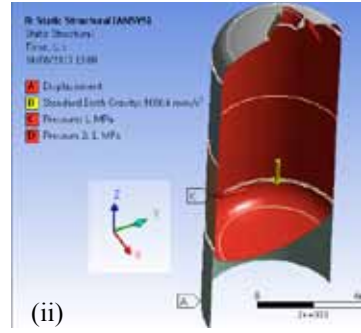
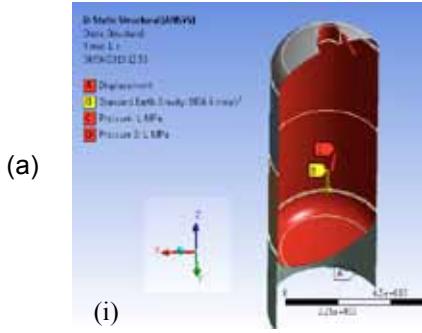


Figure 4 Load Cases

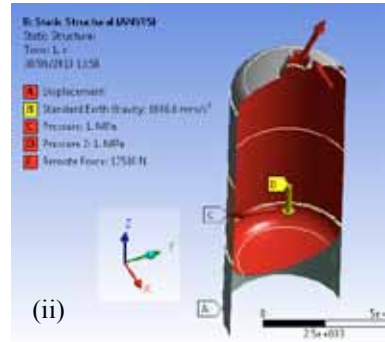
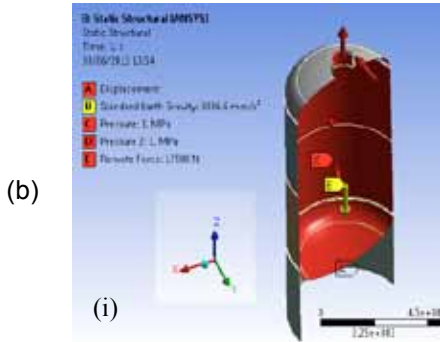
Radial Nozzle

Non-Radial Nozzle

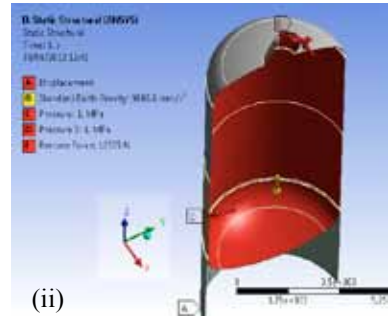
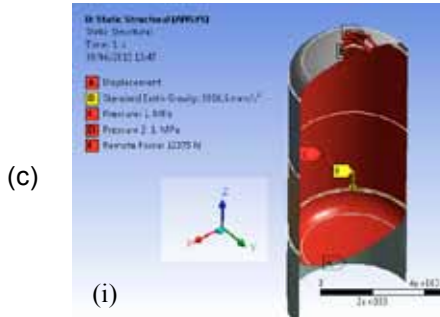
Load Case 1:



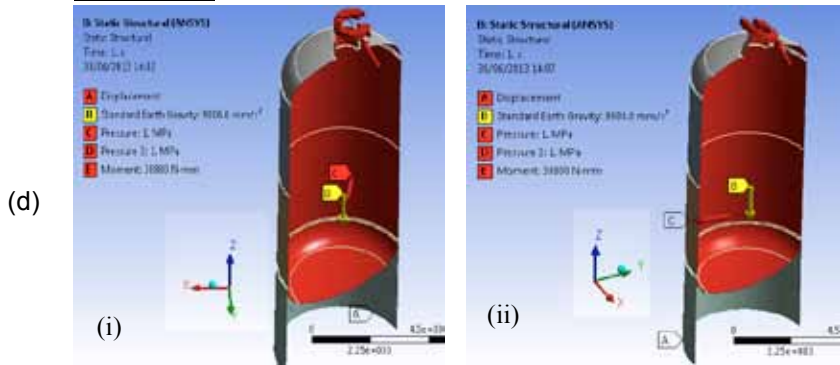
Load Case 2:



Load Case 3:



Load Case 4:



Load Case 5:

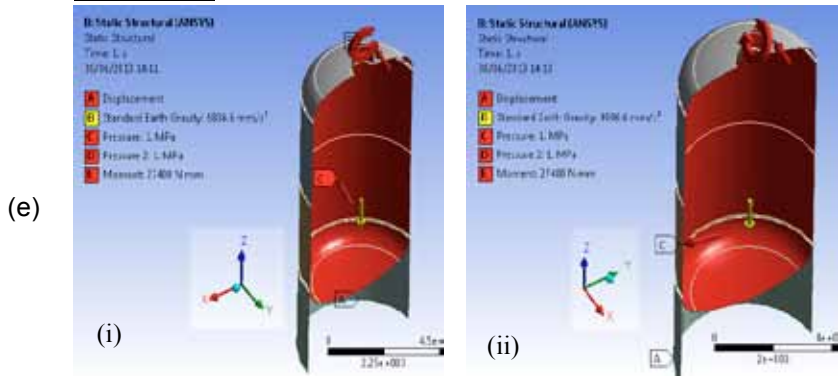


Figure 5 Set up load in FE for Radial and Non-Radial Nozzle  
 (a) Load Case 1, (b) Load Case 2, (c) Load Case 3, (d) Load Case 4 and  
 (e) Load Case 5



## Parametric Study

The true behavior of the vessel could be understood by examining the influence of non-dimensional geometric parameters. The non-dimensional geometric parameters used in this research shows in Table 3. To determine the stress concentration factor (SCF) around the radial and non-radial nozzle attached to the ellipsoidal head, the Equivalent (Von Mises) Stress result for each model must be recorded. SCF is the ratio of max stress on the area under investigation against the actual stress of clean vessel dome due to internal pressure only. This achieved by finding the SCF results for the ratio thickness of the nozzle & head ( $h/H$ ), plotted with respect to the ratio mean diameter of the nozzle & head ( $d/D$ ). Graphs for each load applied produced.

Table 3 Non-dimensional geometric parameters

d/D	h/H	Description	<u>Notation:</u>
0.05	0.48		d = mean diameter of nozzle (250, 500 & 1000 mm)
	0.74		D = mean diameter of ellipsoidal head (5000 mm)
	1		h = thickness of nozzle (12, 18.5 & 25 mm)
0.1	0.48		H = thickness of ellipsoidal head (25 mm)
	0.74		d/D = ratio mean diameter of nozzle & head (0.05, 0.1 & 0.2)
	1		h/H = ratio thickness of nozzle & head (0.48, 0.74 & 1)
0.2	0.48		α = angular parameter of nozzle axis (radial = 0° & non-radial = 30°)
	0.74		
	1		

## Analytical Approach

In order to validate the FEA method, an analytical investigation using thin shell theory was carried out to determine the stress distribution at the radial nozzle due to the internal pressure loading. The FEA method can be categorized as valid if the compared results between the analytical calculations were not exceeding 10%. This analytical approach and FEA method achieved by referring to the previous works [5-20].

The nozzle connections to the ellipsoidal head vessel have two different shells of revolution. The nozzle is cylindrical shell, and head is

ellipsoidal shell. The membrane forces and edge forces acting at the intersection of both shells (refer to: Fig. 6) need to satisfy the equations of compatibility and equilibrium at the junctures. By these, the unknown edge forces and bending moments can be determined. Once these values obtained, the stress resultants at any point in the ellipsoidal shell, can be calculated by superimpose to the membrane solutions.

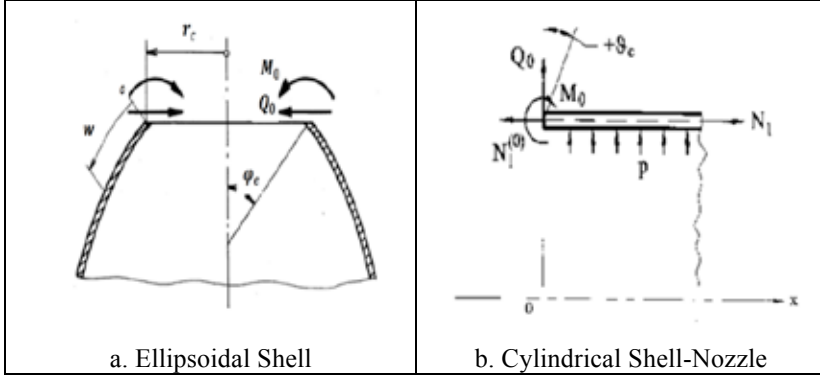


Figure 6 Membrane and Edge Forces of Ellipsoidal Head and Nozzle from the reference work [5]

### Compatibility and Equilibrium At Junctures

Strain continuity at the juncture where cylindrical and ellipsoidal connected depends on equal total displacements and twist angles of the connected parts. In order to achieve inner mechanical balance of two different shells joined, it is necessary for this case to satisfy the boundary conditions expressed by the following equations:

$$\frac{Q_0}{2\beta^2 \cdot D_c} + \frac{M_0}{\beta \cdot D_c} = \frac{M_0}{\alpha \cdot D_e} \cdot R_1 + \frac{Q_0 R_1^2}{2\alpha^2 \cdot D_e} \frac{M_0}{\alpha \cdot D_e} \cdot R_1 + \frac{Q_0 R_1^2}{2\alpha^2 \cdot D_e} \quad (1)$$

$$\frac{r_c}{E \cdot h_c} \cdot (N_{2\theta c} - \nu \cdot N_{1xc}) + \frac{Q_0}{2\beta^3 \cdot D_c} + \frac{M_0}{2\beta^2 \cdot D_c} =$$

$$\frac{r_c}{E \cdot h_e} \cdot (N_{2m\epsilon} - \nu \cdot N_{1\phi\epsilon}) + \frac{R_2 \sin \phi_\epsilon}{E \cdot h_e} \quad (2)$$

$$\left[ M_0 \frac{2R_2 \alpha^4}{R_1^2} + Q_0 \frac{2\alpha}{R_1} + \nu \cdot Q_0 \cot \varphi_\varepsilon \right]$$

$$N_{1xc} \cos \varphi_c + Q_0 \sin \varphi_c = N_{1\varphi\varepsilon} \cos \varphi_\varepsilon + Q_0 \sin \varphi_\varepsilon \quad (3)$$

Where;

$$r_c = \frac{d_c}{2}$$

$$N_{1xc} = \frac{p \cdot r_c}{2}$$

$$N_{2\theta c} = p \cdot r_c$$

$$\varphi_\varepsilon = \sin^{-1} \frac{b}{a} \cdot \frac{q}{\sqrt{1 - \left( \frac{a^2 - b^2}{a^2} \right) q^2}}$$

$$q = \frac{r_c}{a}$$

$$R_1 = \frac{a^2 \cdot b^2}{(a^2 \cdot \sin^2 \varphi_\varepsilon + b^2 \cdot \cos^2 \varphi_\varepsilon)^{\frac{3}{2}}}$$

$$R_2 = \frac{a^2}{(a^2 \cdot \sin^2 \varphi_\varepsilon + b^2 \cdot \cos^2 \varphi_\varepsilon)^{\frac{1}{2}}}$$

$$N_{1\varphi\varepsilon} = \frac{p \cdot R_2}{2 \cdot h_\varepsilon}$$

$$N_{2\theta\varepsilon} = p \cdot R_2 \cdot \left( 1 - \frac{R_2}{2R_1} \right)$$

$$\beta = \sqrt[4]{\frac{3(1 - \nu^2)}{r_c^2 \cdot h_c^2}}$$

$$D_c = \frac{E \cdot h_c^3}{12(1 - \nu^2)}$$

$$\alpha = R_1 \sqrt[4]{\frac{3(1-\nu^2)}{R_2^2 \cdot h_e^2}}$$

$$D_e = \frac{E \cdot h_e^3}{12(1-\nu^2)}$$

**Solution Analytical Calculation.**

Eqs. (1) through (3) used to solve for the unknowns edge force,  $Q_0$  and bending moment,  $M_0$ . Once these values obtained, the stress resultants at any point in the ellipsoidal shell  $w = \varphi - \varphi_e$  can be calculated by superimpose to the membrane solutions as following.

$$N_1 = \cot \varphi \left[ M_0 \frac{2\alpha}{R_1} e^{-\alpha w} \sin \alpha w - Q_0 e^{-\alpha w} (\cos \alpha w - \sin \alpha w) \right] \quad (4)$$

$$N_2 = \frac{2\alpha R_2}{R_1} \left[ M_0 \frac{\alpha}{R_1} e^{-\alpha w} (\cos \alpha w - \sin \alpha w) - Q_0 e^{-\alpha w} \cos \alpha w \right] \quad (5)$$

$$M_1 = M_0 e^{-\alpha w} (\cos \alpha w + \sin \alpha w) + \frac{Q_0 R_1}{\alpha} e^{-\alpha w} \sin \alpha w \quad (6)$$

$$M_2 = \nu M_1 \quad (7)$$

The principal stresses at any point related to the normal stresses in the meridional and circumferential directions are as following:

$$\sigma_1 = \sigma_\varphi = \frac{N_1}{h_e} \pm \frac{6M_1}{h_e^2} \quad (8)$$

$$\sigma_2 = \sigma_\theta = \frac{N_2}{h_e} \pm \frac{6M_2}{h_e^2} \quad (9)$$

$$\sigma_3 = 0 \quad (10)$$

The positive sign applies to points on the inner surface of the shell, and the negative sign applies to points on the outer surface of the shell. In order to validate the FEA results, the principal stresses obtained converted to the Equivalent (Von Mises) Stress as per following equations.

$$V = \frac{1}{\sqrt{2}} \left[ \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} \right] \quad (11)$$

## Results & Discussion

### Validation of Finite Element Analysis

The comparison results between Analytical calculation and FEA method show as per Table 4.

Table 4 Analytical Calculation VS FEA

h/H= 0.48	d/D	Stress at Outer Ellipsoidal Head		Percentage Difference (%)	Stress at Inner Ellipsoidal Head		Percentage Difference (%)
		Analytical Calculation (Mpa)	FEA (Mpa)		Analytical Calculation (Mpa)	FEA (Mpa)	
	0.05	145.07	138.75	-4.35%	214.54	207.13	-3.45%
	0.1	152.25	146.05	-4.07%	210.00	200.44	-4.55%
	0.2	158.65	152.45	-3.91%	199.67	189.67	-5.01%

h/H= 0.74	d/D	Stress at Outer Ellipsoidal Head		Percentage Difference	Stress at Inner Ellipsoidal Head		Percentage Difference
		Analytical Calculation (Mpa)	FEA (Mpa)		Analytical Calculation (Mpa)	FEA (Mpa)	
	0.05	138.90	129.07	-7.08%	212.85	202.04	-5.08%
	0.1	146.69	136.76	-6.77%	206.22	195.26	-5.31%
	0.2	150.70	140.77	-6.59%	192.93	181.97	-5.68%

h/H= 1.0	d/D	Stress at Outer Ellipsoidal Head		Percentage Difference	Stress at Inner Ellipsoidal Head		Percentage Difference
		Analytical Calculation (Mpa)	FEA (Mpa)		Analytical Calculation (Mpa)	FEA (Mpa)	
	0.05	128.73	124.54	-3.25%	197.05	192.54	-2.29%
	0.1	136.45	132.26	-3.07%	190.92	186.46	-2.34%
	0.2	139.36	135.17	-3.01%	177.46	173.00	-2.51%

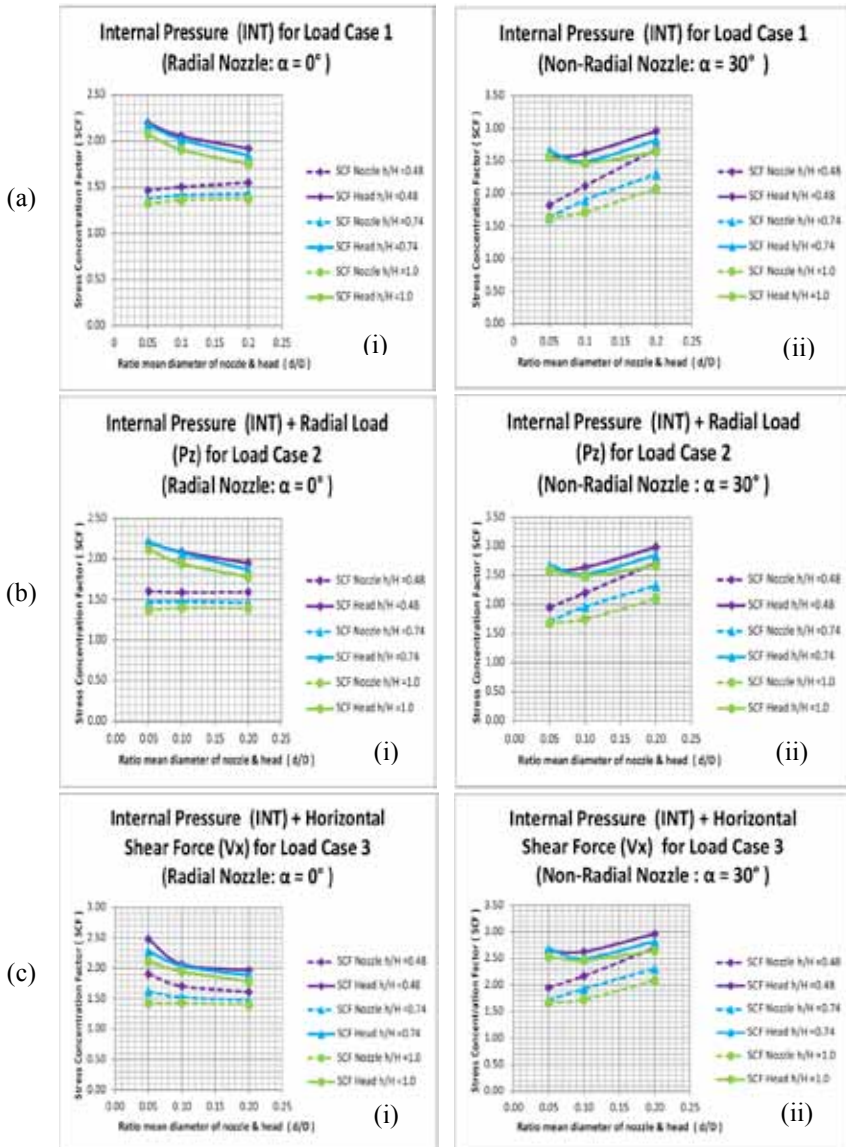
The comparison results between the FEA and analytical calculation demonstrated that the differences were less than 10%, and was quite good for all the analysis. As a conclusion, the finite element model used in this research was valid.

**Results of Parametric Analysis For Radial & Non-Radial Nozzle**

All the results of parametric analysis for radial and non-radial nozzle due to the non-dimensional geometric parameters for each load cases using FEA method show in Fig. 7(a) – (e).

Results of Radial Nozzle

Results of Non-Radial Nozzle



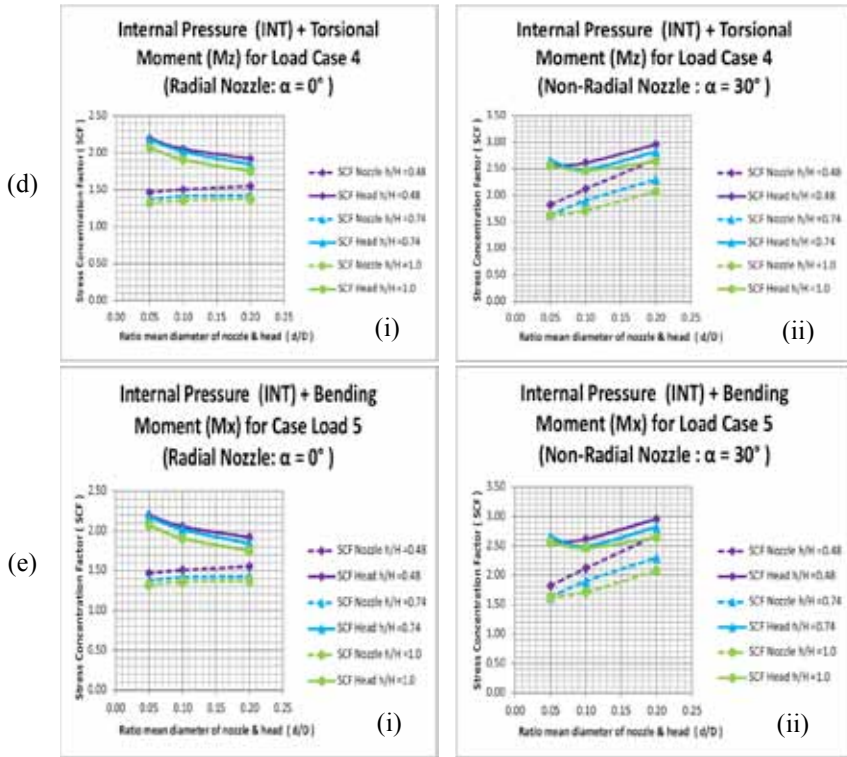


Figure 7 Results of parametric analysis for Radial and Non-Radial Nozzle  
 (a) Load Case 1, (b) Load Case 2, (c) Load Case 3, (d) Load Case 4 and  
 (e) Load Case 5

Figure 7(a)(i)-(ii) illustrates the graph plot for internal pressure for both radial and non-radial nozzle with respect to SCF and ratio mean diameter ( $d/D$ ) of the nozzle and ellipsoidal head. In figure 7(a)(i), with the increase of ratio mean diameter, the value of SCF for thickness  $h/H = 0.48$  at the ellipsoidal head shows to decrease while at the nozzle the SCF increases. It is expected due to the increase of thickness from  $h/H = 0.48$  to  $h/H = 1.0$ , the SCF for both nozzle and head decreased, and it shows from the graphs. However, in figure 7(a)(ii) the graph shows an opposite pattern from the radial nozzle. When the orientation of nozzle rotated to  $\alpha = 30^\circ$ , the graph pattern for both at ellipsoidal head and nozzle show a dramatic increase of SCF when the ratio mean diameter become higher.

From observation due to different load cases, the graphs pattern are quite similar for both radial and non radial except for the load case 3 in figure

7(c)(i). At the ellipsoidal head and nozzle, the value of SCF tends to decrease with the increase of ratio mean diameter, respectively. All these results show that the orientation angle ( $\alpha$ ), size and thickness of nozzle affect the Stress Concentration Factor (SCF).

## **Conclusion**

These parametric study simulations using the FEA method for radial and non-radial nozzle connections in an ellipsoidal head vessel has discovered useful information. All these results will help the designer to increase accuracy, enhance the design and better insight into critical design parameters while designing the pressure vessel. The influence of the orientation angle ( $\alpha$ ), size and thickness of the nozzle on the Stress Concentration Factor (SCF) for the radial and non-radial nozzle connections in ellipsoidal head vessel due to the internal pressure and various external loadings successfully proved.

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