Thermal Buckling of Laminated Plates using Modified Mantari Function

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ABSTRACT

Critical buckling temperature of laminated plate under thermal load varied linearly along the thickness, is developed using a higher-order shape function which depends on a parameter ''m'', which is improved to obtain results for thin and thick plates. Laminated plates' equations of motion are obtained using virtual work principle and solved for simply supported boundary conditions. Angle and cross laminates thermal buckled mode shapes with different E_1/E_2 *proportion, number of plies,* (α_2/α_1) *proportion, aspect ratios, are investigated. It is observed that this shape function gives thermal buckling for thin and thick plates but with m = 0.05 that agree well with other theories and linear distribution of temperature gives a rise to critical temperature approach to 50% than those caused by uniform thermal distribution.*

Keywords: *Linear Thermal Load; Non-uniform Thermal Load; Critical Temperature; Angle-ply Plate; Cross-ply Plate; Shear Deformation Theory*

Introduction

Thermal buckling is an important failure mode for composite plate and shell when worked in thermal environment such as rockets, marine and space vehicles, aerospace structure and launch vehicles may be exposed to thermal loading due to the aerodynamic heating.as well as in other fields of modern engineering structures. Many researchers studied thermal buckling using different theories, therefore investigation of plate thermal buckling is of special interest.

Critical temprature of plate made from composite laminates is obtained using numerical methods such as differential quadrature method (DQM) [1]-

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[2] and finite element method [3]-[7], these plates may be under uniform and/or non-uniform, linear or nonlinear thermal load with variuos boundary conditions, others [8]-[10] used variation methods to investigate buckling of laminated plates in thermal environment especialy with different boundary conditions. Fazzolari and Carrera [11] and Fazzolari [12] carried out thermal stability and free vibration analyses of functionally graded (FG) and sandwich plates using Ritz and advanced Ritz method, taking into consideration uniform, linear and non-linear temperature rises through the-thickness layer plate. Xing and Wang [13], investigated thermal buckling for rectangular plates made from functionally graded material, under different boundary conditions using separation-of-variable method. Karami et al. [14], modified 3D plate theory to disperse wave in functionally graded nanoplates in hygro thermal environment and rest on an elastic foundation, Nicholas et al. [15] used genetic algorithm to maximize the critical temperature of the laminated composite plate subjected to different nonuniform thermal loads, therefore ply angle and stacking sequence are optimized.

Many researchers developed and used different higher-order shear deformation theory (HSDT) to study bending, buckling and response of plate with simply supported boundary [16]-[17], on a viscoelastic foundation or resting on a Winkler-Pasternak elastic foundation, in thermal or in a hygro-thermal environment or under hygro-thermo-mechanical load [18]–[20] which were as composite, functionally graded (FG), sandwich plate or made from porous functionally graded material [21]-[22], while effect of these enviornment on vibration behavior for these plates were investigated by [23]-[24]. Thermal stability of porous functionally graded material beam (of power law) under different thermal load distribution and boundary conditions were analyzed by [25]. Effect of reinforcement material (such as carbon nanotubes (CNTs)) on vibration berhavior of microplates and shell in thermal environment were discussed by [26]-[27], using developed shear deformation theory with five or more unknown.

Different simple refined higher order shear deformation theory were developed and used to analyze thermo-mechanical buckling [28]-[31], bending and dynamic analyses of composite plates as in [32]-[34].

In present work, high order shear deformation theory is used to obtain critical temperature of simply supported laminated composite plate under linearly distributed temperature along the thickness, using displacement field proposed by Mantari et al. [35] but with m = 0.05 as developed by [36], and used it by [37] to obtain thermal buckling of laminated plates under unifrom thermal load. Effect of many design parameters for thin and thick plate, E_1/E_2 ratio, aspect ratio, α_2/α_1 ratio for symmetric and antisymmetric cross and angle ply are investigated.

Displacements and strain

In present work, developed Mantari's displacement field with m = 0.05 value is used to obtain critical thermal temperature of simply supported laminated plate, this modified field of displacement is, [36]:

$$\bar{u}(x,y,z) = u_x(x,y) + z\left(\frac{m\pi}{h}\theta_x - \frac{\partial w}{\partial x}\right) + \left(e^{m\cos\left(\frac{\pi z}{h}\right)} * sin\frac{\pi z}{h}\right) * \theta_x \quad (1)$$

$$\bar{v}(x,y,z) = v_y(x,y) + z\left(\frac{m\pi}{h}\theta_y - \frac{\partial w}{\partial y}\right) + \left(e^{m\cos\left(\frac{\pi z}{h}\right)} * sin\frac{\pi z}{h}\right) * \theta_y \quad (2)$$

$$\bar{w}(x,y,z) = w(x,y) \quad (3)$$

with: m = 0.05.

Linear strain based on displacement field are:

$$\varepsilon_{xx} = \varepsilon^0_{xx} + z\varepsilon^1_{xx} + \sin\frac{\pi z}{h} e^{m\cos\left(\frac{\pi z}{h}\right)}\varepsilon^2_{xx} \quad (4)$$

$$\varepsilon_{yy} = \varepsilon^0_{yy} + z\varepsilon^1_{yy} + \sin\frac{\pi z}{h} e^{m\cos\left(\frac{\pi z}{h}\right)}\varepsilon^2_{yy} \quad (5)$$

$$\gamma_{xy} = \varepsilon^0_{xy} + z\varepsilon^1_{xy} + \sin\frac{\pi z}{h} e^{m\cos\left(\frac{\pi z}{h}\right)}\varepsilon^2_{xy} \quad (6)$$

$$\gamma_{xz} = \varepsilon^0_{xz} + (-m * \sin^2(\frac{\pi z}{h}) + \cos\frac{\pi z}{h})\frac{\pi}{h} e^{m\cos\left(\frac{\pi z}{h}\right)}\varepsilon^3_{xz} \quad (7)$$

$$\gamma_{yz} = \varepsilon^0_{yz} + (-m * \sin^2(\frac{\pi z}{h}) + \cos\frac{\pi z}{h})\frac{\pi}{h} e^{m\cos\left(\frac{\pi z}{h}\right)}\varepsilon^3_{yz} \quad (8)$$

where:

$$\begin{Bmatrix} \varepsilon^0_{xx} \\ \varepsilon^0_{yy} \\ \gamma^0_{xy} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u_x}{\partial x} \\ \frac{\partial v_y}{\partial x} \\ \frac{\partial u_x}{\partial y} + \frac{\partial v_y}{\partial x} \end{Bmatrix}, \begin{Bmatrix} \varepsilon^1_{xx} \\ \varepsilon^1_{yy} \\ \gamma^1_{xy} \end{Bmatrix} = \begin{Bmatrix} \frac{m\pi}{h}\frac{\partial\theta_x}{\partial x} - \frac{\partial^2 w}{\partial x^2} \\ \frac{m\pi}{h}\frac{\partial\theta_y}{\partial y} - \frac{\partial^2 w}{\partial y^2} \\ \frac{m\pi}{h}\frac{\partial\theta_y}{\partial x} + \frac{m\pi}{h}\frac{\partial\theta_x}{\partial y} \\ -2\frac{\partial^2 w}{\partial x\,\partial y} \end{Bmatrix} \quad (9, 10)$$

$$\begin{Bmatrix} \varepsilon^2_{xx} \\ \varepsilon^2_{yy} \\ \gamma^2_{xy} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial\theta_x}{\partial x} \\ \frac{\partial\theta_y}{\partial y} \\ \frac{\partial\theta_y}{\partial x} + \frac{\partial\theta_x}{\partial y} \end{Bmatrix} \quad (11)$$

$$\begin{Bmatrix}\Gamma_{xz}^{0}\\ \gamma_{yz}^{0}\end{Bmatrix}=\begin{Bmatrix}m\frac{\pi}{h}\Theta_x\\ m\frac{\pi}{h}\Theta_y\end{Bmatrix},\quad \begin{Bmatrix}\gamma_{xz}^{3}\\ \gamma_{yz}^{3}\end{Bmatrix}=\begin{Bmatrix}\Theta_x\\ \Theta_y\end{Bmatrix} \qquad \text{(12 and 13)}$$

Equations of motion

To derive equations of motion, principle of virtual displacements is used with new shape function Reddy [38].

$$0=\int_0^t \delta U+\delta V \qquad (14)$$

For which:

$$\delta U=\int_{\frac{-h}{2}}^{\frac{h}{2}}\{\int_{\Omega}^{k}[\sigma_{xy}\delta\varepsilon_{xy}^{k}+\sigma_{yz}\delta\varepsilon_{yz}^{k}+\sigma_{xz}\delta\varepsilon_{xz}^{k}+\sigma_{xx}\delta\,\varepsilon_{xx}^{k}+\sigma_{yy}\delta\varepsilon_{yy}^{k}]\ \partial x\,\partial y\}\,\partial z=0 \qquad (15)$$

$$\delta U=\int(N_1\delta\,\varepsilon_{xx}^{0}+N_2\delta\varepsilon_{yy}^{0}+N_6\delta\varepsilon_{Xy}^{0}+M_1\delta\varepsilon_{xx}^{1}+M_2\delta\varepsilon_{yy}^{1}+M_6\delta\varepsilon_{Xy}^{1}+P_1\delta\varepsilon_{xx}^{2}+P_2\delta\varepsilon_{yy}^{2}+P_6\delta\varepsilon_{Xy}^{2}+Q_2\delta\varepsilon_{yz}^{0}+Q_1\delta\varepsilon_{xz}^{0}+K_1\delta\varepsilon_{xz}^{3}+K_2\delta\varepsilon_{yz}^{3})\,\partial x\,\partial y=0 \qquad (16)$$

where, (N_i, Mi, Pi, Qi and K_i) are the load results from the following integration:

$$(N_i, M_i, P_i)=\sum_{k=1}^{N}\int_{z^{k-1}}^{z^{k}}\left(1\,,z,\sin\left(\frac{\pi z}{h}\right)*e^{m\cos\left(\frac{\pi z}{h}\right)}\right)*\sigma_i^k*dz \qquad (i=1,2,6)$$

$$(Q_1,\ K_1)=\sum_{k=1}^{N}\int_{z^{k-1}}^{z^{k}}\left(1,\frac{\pi}{h}\left(-m*\ \sin^2\left(\frac{\pi z}{h}\right)+\cos\left(\frac{\pi z}{h}\right)\right)*e^{m\cos\left(\frac{\pi z}{h}\right)}\right)*\sigma_5^k*dz$$

$$(Q_2,\ K_2)=\sum_{k=1}^{N}\int_{z^{k-1}}^{z^{k}}\left(1,\frac{\pi}{h}\left(-m*\ \sin^2\left(\frac{\pi z}{h}\right)+\cos\left(\frac{\pi z}{h}\right)\right)*e^{m\cos\left(\frac{\pi z}{h}\right)}\right)*\sigma_4^k*dz$$

Substituting the virtual strain from Equations (4-13) into Equation (16) to get:

$$0=-\int[\frac{\partial N_1}{\partial x}\ \delta u_x+\frac{\partial N_6}{\partial y}\delta u_x-\frac{\partial^2 M_1}{\partial x^2}\delta w+2\frac{\partial^2 M_6}{\partial x\,\partial y}\delta w+\frac{m\pi}{h}\frac{\partial M_2}{\partial y}\delta\Theta_y-\frac{\partial^2 M_2}{\partial y^2}\delta w+\frac{\partial N_2}{\partial y}\delta v_y+\frac{\partial N_6}{\partial x}\delta v_y+\frac{m\pi}{h}\ \frac{\partial M_6}{\partial y}\delta\Theta_x+\frac{m\pi}{h}\frac{\partial M_1}{\partial x}\ \delta\Theta_x+\frac{\partial P_6}{\partial y}\delta\Theta_x+\frac{\partial P_1}{\partial x}\delta\Theta_x-\frac{m\pi}{h}Q_1\delta\Theta_x-K_1\delta\Theta_x+(\frac{m\pi}{h}\ \frac{\partial M_6}{\partial x})\delta\Theta_y+\frac{\partial P_6}{\partial x}\delta\Theta_y+\frac{\partial P_2}{\partial y}\delta\Theta_y-(\frac{m\pi}{h}Q_2)\delta\Theta_y\ -K_2\delta\Theta_y]\,\partial x\,\partial y \qquad (17)$$

Virtual work of thermal applied load δV is:

$$\delta V = \int_{\Omega} \left\{ N_x^T \,\delta\left(\frac{\partial\ w}{\partial x}\right)^2 + N_y^T \,\delta\left(\frac{\partial\ w}{\partial y}\right)^2 \right\} dxdy \tag{18}$$

Five equations of motion are derived by substituting Equations (17) and (18) into Equation (16):

$$\delta u: \frac{\partial N_{12}}{\partial y} + \frac{\partial N_1}{\partial x} = 0 \tag{19}$$

$$\delta v: \frac{\partial N_{12}}{\partial x} + \frac{\partial N_2}{\partial y} = 0 \tag{20}$$

$$\delta w: 2\frac{\partial^2 M_{12}}{\partial x\,\partial y} + \frac{\partial^2 M_1}{\partial x^2} + \frac{\partial^2 M_2}{\partial y^2} + N_x^T \left(\frac{\partial^2 w}{\partial x^2}\right) + N_y^T \left(\frac{\partial^2 w}{\partial y^2}\right) = 0 \tag{21}$$

$$\delta \Theta x: \frac{m\pi}{h}\frac{\partial M_1}{\partial x} + \frac{m\pi}{h}\frac{\partial M_{12}}{\partial y} + \frac{\partial P_1}{\partial x} + \frac{\partial P_{12}}{\partial y} - \frac{m\pi}{h}Q_1 - K_1 = 0 \tag{22}$$

$$\delta \Theta y: \frac{m\pi}{h}\frac{\partial M_2}{\partial y} + \frac{m\pi}{h}\frac{\partial M_{12}}{\partial x} + \frac{\partial P_2}{\partial y} + \frac{\partial P_{12}}{\partial x} - \frac{m\pi}{h}Q_2 - K_2 = 0 \tag{23}$$

An orthotropic lamina stress-strain relation for a plane state of stress is [38]:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{11} Q_{12} Q_{16} \\ Q_{12} Q_{22} Q_{26} \\ Q_{16} Q_{26} Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} - \alpha_{xx}\Delta T \\ \varepsilon_{yy} - \alpha_{xx}\Delta T \\ \gamma_{xy} - 2\alpha_{xy}\Delta T \end{Bmatrix}, \begin{Bmatrix} \sigma_{yz} \\ \sigma_{xz} \end{Bmatrix} = \begin{bmatrix} Q_{44} & Q_{45} \\ Q_{45} & Q_{55} \end{bmatrix} \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} \tag{24}$$

The force results- strains related as:

$$\begin{Bmatrix} N_1 \\ N_2 \\ N_{12} \\ M_1 \\ M_2 \\ M_{12} \\ P_1 \\ P_2 \\ P_{12} \end{Bmatrix} = \begin{bmatrix} A_{11} A_{12} A_{16} & B_{11} B_{12} B_{16} & E_{11} E_{12} E_{16} \\ A_{12} A_{22} A_{26} & B_{12} B_{22} B_{26} & E_{12} E_{22} E_{26} \\ A_{16} A_{26} A_{66} & B_{16} B_{26} B_{66} & E_{16} E_{26} E_{66} \\ B_{11} B_{12} B_{16} & D_{11} D_{12} D_{16} & F_{11} F_{12} F_{16} \\ B_{12} B_{22} B_{26} & D_{12} D_{22} D_{26} & F_{12} F_{22} F_{26} \\ B_{16} B_{26} B_{66} & D_{16} D_{26} D_{66} & F_{16} F_{26} F_{66} \\ E_{11} E_{12} E_{16} & F_{11} F_{12} F_{16} & H_{11} H_{12} H_{16} \\ E_{12} E_{22} E_{26} & F_{12} F_{22} F_{26} & H_{12} H_{22} H_{26} \\ E_{16} E_{26} E_{66} & F_{16} F_{26} F_{66} & H_{16} H_{26} H_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \varepsilon_{xy}^0 \\ \varepsilon_x^1 \\ \varepsilon_y^1 \\ \varepsilon_{xy}^1 \\ \varepsilon_x^2 \\ \varepsilon_y^2 \\ \varepsilon_{xy}^2 \end{Bmatrix} \tag{25}$$

$$\begin{Bmatrix} Q_1 \\ Q_2 \\ K_1 \\ K_2 \end{Bmatrix} = \begin{bmatrix} A_{44} & A_{45} & J_{44} & J_{45} \\ A_{45} & A_{55} & J_{45} & J_{55} \\ J_{44} & J_{45} & L_{44} & L_{45} \\ J_{45} & J_{55} & L_{45} & L_{55} \end{bmatrix} \begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \\ \gamma_{yz}^3 \\ \gamma_{xz}^3 \end{Bmatrix} \tag{26}$$

$$\begin{Bmatrix} N_x^T \\ N_y^T \end{Bmatrix} = \sum_{k=1}^{N} \int_{z^k}^{z^{k+1}} \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{12} & Q_{22} & Q_{26} \end{bmatrix} \begin{Bmatrix} \alpha_{xx} \\ \alpha_{yy} \\ 2\alpha_{xy} \end{Bmatrix} \Delta T dz \tag{27}$$

$$\begin{Bmatrix} M_x^T \\ M_y^T \end{Bmatrix} = \sum_{k=1}^{N} \int_{z^k}^{z^{k+1}} \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{12} & Q_{22} & Q_{26} \end{bmatrix} \begin{Bmatrix} \alpha_{xx} \\ \alpha_{yy} \\ 2\alpha_{xy} \end{Bmatrix} \Delta T z dz \tag{28}$$

where:

$$(A_{ij}, B_{ij}, D_{ij}, E_{ij}) = \int_{\frac{-h}{2}}^{\frac{h}{2}} Q_{ij}(1, z, z^2, \sin(\frac{\pi z}{h}) e^{m\cos\left(\frac{\pi z}{h}\right)} dz \quad (i, j=1, 2, 6) \tag{29}$$

$$(F_{ij}, H_{ij}) = \int_{\frac{-h}{2}}^{\frac{h}{2}} Q_{ij}\, (z * \sin\left(\frac{\pi z}{h}\right) e^{m\cos\left(\frac{\pi z}{h}\right)}, e^{2m\cos\left(\frac{\pi z}{h}\right)} * \sin^2(\frac{\pi z}{h})\,)$$

$$(i, j=1, 2, 6) \tag{30}$$

$$J_{ij} = \int_{\frac{-h}{2}}^{\frac{h}{2}} Q_{ij} \frac{\pi}{h} e^{m\cos\left(\frac{\pi z}{h}\right)} \left(-m * \sin^2\left(\frac{\pi z}{h}\right) + \cos\left(\frac{\pi z}{h}\right)\right) * dz \tag{31}$$

$$L_{ij} = \int_{\frac{-h}{2}}^{\frac{h}{2}} Q_{ij} \left(\frac{\pi}{h}\right)^2 e^{2m\cos\left(\frac{\pi z}{h}\right)} \left(-m * \sin^2\left(\frac{\pi z}{h}\right) + \cos\frac{\pi z}{h}\right)^2 dz \quad (i, j = 4, 5) \tag{32}$$

while $\Delta T = \frac{\Delta T}{h}\left(z + \frac{h}{2}\right)$ and Qij (i, j=1, 2, 4, 5, 6) = plane stress reduced stiffness.

Navier solution

Navier's series for simply supported cross laminates, are [38]:

$$u_x(x, y, t) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \overline{U}_{mn}(t) * \cos\left(\frac{m\pi}{a} x\right) * \sin\left(\frac{n\pi}{b} y\right) \tag{33}$$

$$v_y(x, y, t) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \overline{V}_{mn}(t) * \sin\left(\frac{m\pi}{a} x\right) * \cos\left(\frac{n\pi}{b} y\right) \tag{34}$$

$$w(x, y, t) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \overline{W}_{mn}(t) * \sin\left(\frac{m\pi}{a} x\right) * \sin\left(\frac{n\pi}{b} y\right) \tag{35}$$

$$\theta_x(x,y,t) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \overline{\theta_x}_{mn}(t) * \cos\left(\frac{m\pi}{a}x\right) * \sin\left(\frac{n\pi}{b}y\right) \tag{36}$$

$$\theta_y(x,y,t) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \overline{\theta_y}_{mn}(t) * \sin\left(\frac{m\pi}{a}x\right) * \cos\left(\frac{n\pi}{b}y\right) \tag{37}$$

While for simply supported Angle-Ply laminates, the following series are used [37]:

$$u(x,y,t) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \overline{U}_{mn}(t) * \sin\left(\frac{m\pi}{a}x\right) * \cos\left(\frac{n\pi}{b}y\right) \tag{38}$$

$$v(x,y,t) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \overline{V}_{mn}(t) * \cos\left(\frac{m\pi}{a}x\right) * \sin\left(\frac{n\pi}{b}y\right) \tag{39}$$

$$w(x,y,t) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \overline{W}_{mn}(t) * \sin\left(\frac{m\pi}{a}x\right) * \sin\left(\frac{n\pi}{b}y\right) \tag{40}$$

$$\theta_x(x,y,t) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \overline{\theta_x}_{mn}(t) * \sin\left(\frac{m\pi}{a}x\right) * \cos\left(\frac{n\pi}{b}y\right) \tag{41}$$

$$\theta_y(x,y,t) = \sum_{n=1}^{\infty}\sum_{m=1}^{\infty} \overline{\theta_y}_{mn}(t) * \cos\left(\frac{m\pi}{a}x\right) * \sin\left(\frac{n\pi}{b}y\right) \tag{42}$$

where: a, b are dimensions of plate.

Eignvalue problem

Equations of motion as function of displacement componants are obtained by subtituting force and moment resultants from Equations (25 - 28) in Equations (19-23) and then by substituting Equations (33-37) for cross ply and Equations (38-42) for angle ply, the following eignvalue problem is obtained:

$$\left(\begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} & k_{15} \\ & k_{22} & k_{23} & k_{24} & k_{25} \\ & & k_{33-\alpha^2(N_x^T+M_x^T)-\beta^2(N_y^T+M_y^T)-\alpha\beta N_{xy}^T} & k_{34} & k_{35} \\ & & & k_{44} & k_{45} \\ & & & & k_{55} \end{bmatrix}\right)\{e\} = 0 \tag{43}$$

where: $\{e_{ij}\} = \left\{U_{mn}, V_{mn}, W_{mn},\ \theta_{x_{mn}}, \theta_{y_{mn}}\right\}$, and k_{ij} is element of stiffness, from Equation (43) thermal buckling of the plate is calculated.

Results and Discussion

Critical temperature for angle and cross laminated plates under non-uniform temperature distribution is obtained by programming present solution using MATLAB15. To verify the suggested above solution, the obtained results compared with other works as shown in Table 1 and Table 2, which agree well. The effect of design parameters on critical temperature of laminated plates are analyzed such as, symmetric and antisymmetric ply, number of layers, orthotropy ratio E_1/E_2, thermal expansion coefficient ratio (α_2/α_1) under uniform and linear temperature distribution.

Table 1: Normalized critical temperature for isotropic square plate, E1 = E2 = 380 Gpa, ν = 0.3, α = 7.4e-6

a/H	100	80	60	40	20	10
Present	31.15	48.67	86.55	194.479	771.269	2961.0484
Cetkovic [4]	34.182	53.395	94.871	213.113	845.027	3266.311
CLPT	24.198	43.434	84.995	203.738	844.955	3409.821
HODT	24.177	43.387	84.484	202.984	833.032	3224.968

Table 2: Critical temperature (Tcr = T*1e-6) for isotropic square plate (a/H = 100), E = 1, ν = 0.3, α = 2e-6

Tcr			
Present work	**Cetkovic [4]**	**Kari et al. [39]**	**Discrepency%**
115.256	126.5	126	9.5

Critical buckling temperature of thick plate is larger than those for thin plate while uniform temperature distribution cause plate buckle under smaller temperature than those under linear distribution as shown in Table 3. Symmetric and antisymmetric cross ply $[0/90]_2$ and $[0/90]_4$ for square plate with different thickness ratio, are obtained in Table 4, as expected that critical temperature increases as number of layer increases, but it decreases with increasing thicknes ratio and it is larger for antisymmetric, for following tables and figures the layers properties used are: $E_1/E_2 = 25$, $G_{12} = G_{13} = 0.5\ E_2$, $G_{23} = 0.2\ E_2$, $\nu_{12} = \nu_{13} = \nu_{23} = 0.25$, $\alpha_2/\ \alpha_1 = 3$.

Normalized critical temperature for atisymmetric angle ply thick and thin square plate are listed in Table 5, which shows that increasing (a/H) ratio decreases critical temperature. Different critical thermal buckling modes for plates for $[30/-30]_4$ angle-ply square plate are shown in Figures 1 and 2. For these four figures the dimensionless critical temperature is ($T^*cr = T*a^2*h/\pi^2*D_{22}$).

Table 3: Critical temperature Tcr for cross-ply square plate

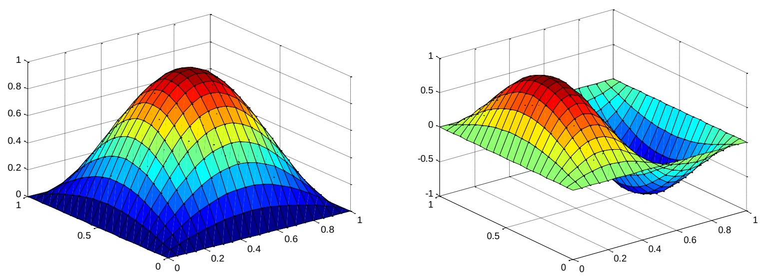
a/H	Tcr		Discrepancy %
	Uniform	**Linear**	
4	0.0109699	0.022893	52
10	0.002874	0.005846	50.8
20	0.0007931	0.00159957	50.4

Table 4: Effect of lamine type on critical temperature Tcr for cross-ply square plate

a/H	Symmetric cross-ply		Anti-Symmetric cross-ply	
	$(0/90)_2$	$(0/90)_4$	$(0/90)_2$	$(0/90)_4$
5	0.0197	0.02431	0.02327	0.02501
10	0.009699	0.01120	0.01018	0.01124
50	0.0006238	0.0006303	0.00053968	0.0006088

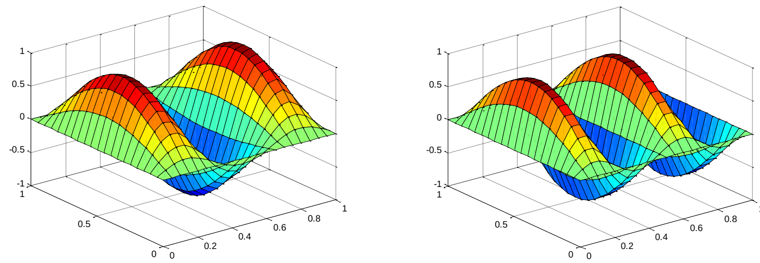
Table 5 Normalized critical temperature [$Tcr = T^* \alpha_1 * 10 * (b/H)^2$] for angle-ply square plate

a/H	Tcr		
	$[30/-30]_4$	**$[45/-45]_4$**	**$[60/-60]_4$**
5	7.3041	7.5790	7.3041
10	15.4472	16.5984	15.4472
50	24.5405	27.5317	24.5405



(a) (b)

Figure 1: Normalized thermal buckling mode of angle-ply $[\pm 30]_4$ square plate, a) mode shape (m = 1, n = 1), b) mode shape (m = 2, n = 1)



(a) (b)

Figure 2: Normalized thermal buckling mode of angle-ply $[\pm 30]_4$ square plate, a) mode shape (m = 3, n = 1), b) mode shape (m = 4, n = 1)

Effect of changing (E_1/E_2) on critical temperature for eight layers anti symmetric angle ply plates for different angle ply are shown in Figure 3, as expexted when increasing orthotropy ratio will increases buckling temprature (it seems reversely because Tcr is devided by D_{22}), also effect of thermal expansion coefficient ratio (α_2/α_1) on critical buckling temperature of eight layer laminated, are obtained in Figure 4 as expected critical temperature decreases when (α_2/ α_1) increase since it decreases stiffness of plate, and angle plies plate has thermal buckling behavior better than cross plate, thickness and aspect ratio are (a/H = 10, a/b = 1).

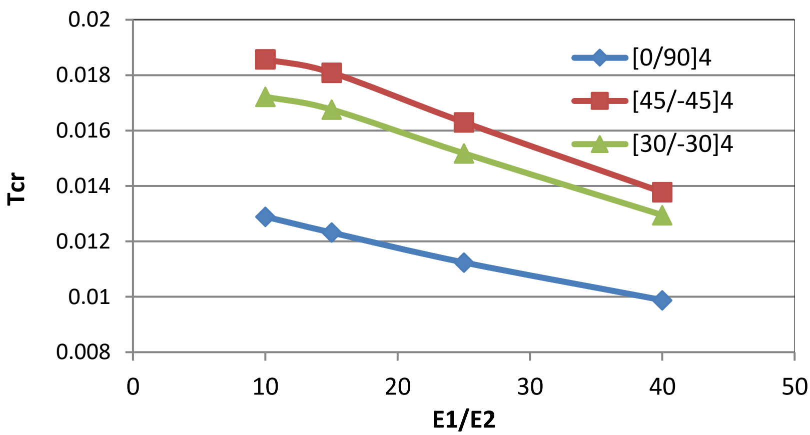


Figure 3: Normalized thermal buckling mode for antisymmetric plate, mode shape (m = 1, n = 1)

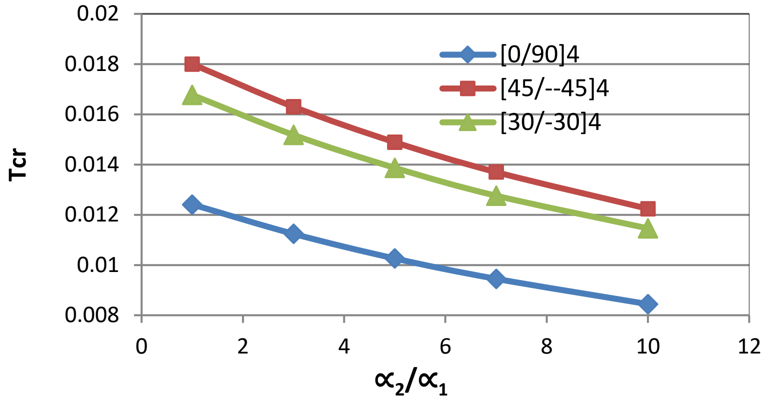


Figure 4: Normalized thermal buckling mode for antisymmetric plate, mode shape (m = 1, n = 1)

Maximum critical buckling temperature is for plates with (45°) angle as listed in Table 6, and it is greater for plate under linear than uniform temperature distribution.

Table 6: Normalized critical temperature [$Tcr = T^* \alpha_1 * 10 * (b/H)^2$] for angle-ply square plate

Angle	No. of layers	Tcr		Discrepancy %
		Linear	Uniform [40]	
	2	9.6671	4.8461	49.8
30	4	19.3646	9.5515	50.6
	8	21.7228	10.6972	50.7
	2	10.0923	5.0503	48
45	4	21.3294	10.5094	50.7
	8	24.0398	11.8277	50.7
	2	9.6671	4.8461	49.8
60	4	19.3646	9.5515	50.6
	8	21.7228	10.6972	50.7

Conclusions

Mantari displacement field but with 'm = 0.05' developed by Majeed and Sadiq [36], is used to analyze critical buckling temperature of cross and angle laminated composite thick and thin plates, under linear distribution of temperature and gives results agree well with those obtained by other theories. As expected critical temperature decreases as thickness ratio and thermal expansion coefficient ratio (α_2/α_1) increase while it increases with the increase of aspect ratio and orthotropy ratio, and angle plies plate has thermal buckling behavior better than cross plate, while critical temperature of laminated plates under linear distribution of temperature along thickness is higher than that under uniform thermal distribution approach (50%), also the thermal buckling mode of these plates are not change.

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References

[1] M. Mansour Mohieddin Ghomshei and Amin Mahmoudi, “Thermal buckling analysis of cross-ply laminated rectangular plates under nonuniform temperature distribution: A differential quadrature approach”, *Journal of Mechanical Science and Technology*, vol. 24, no.12, pp. 2519-2527, 2010.

[2] Guiju Zhang, Caiyuan Xiao, Alireza Rahimi and Mehran Safarpour, “Thermal and mechanical buckling and vibration analysis of FG-GPLRC annular plate using higher order shear deformation theory and generalized differential quadrature method”, *International Journal of Applied Mechanics*, vol. 12, no. 2, pp. 1-28, 2020.

[3] Le-Chung Shiau, Shih-Yao Kuo and Cheng-Yuan Chen, “Thermal buckling behavior of composite laminated plates”, *Composite Structures*, vol. 92, pp. 508–514, 2010.

[4] M. Cetkovic and Lénárt György, “Thermo-Elastic stability of angle-ply laminates application of layerwise finite elements”, *Structural Integrity And Life*, vol. 16, no. 1, pp. 43–48, 2016.

[5] M. Cetkovic, “Thermal buckling of laminated composite plates using layerwise displacement model”, *Composite Structures*, vol. 142, pp. 238-253, 2016.

[6] M.S. Meon, N.H. Mohamad Nor, S. Shawal and J.B. Saedon, “On the Modelling Aspect of Low Velocity Impact Composite Laminates”, *Journal of Mechanical Engineering*, vol.17, no.2, pp.13-25, 2020.

[7] Farah Abdoun & Lahcen Azrar, “Thermal buckling and vibration of laminated composite plates with temperature dependent properties by an asymptotic numerical method”, *International Journal for Computational Methods in Engineering Science and Mechanics*, 2020, doi:10.1080/15502287.2020.1729899.

[8] Wael Abdul-Majeed, Muhsin J. Jweeg and Adnan N. Jameel, "Thermal buckling of rectangular plates with different temperature distribution using strain energy method", *Journal of Engineering*, vol. 17, no. 5, pp. 1047-1065, 2011.

[9] Chun-Sheng Chen, Wei-Ren Chen and Hung-Wei Lin, "Thermally induced stability and vibration of initially stressed laminated composite plates", *Mechanika*, vol. 22, no. 1, pp. 51-58, 2016.

[10] R. Vescovini, M. D'Ottavio, L. Dozio, O. Polit, "Thermal Buckling Response of Laminated and Sandwich Plates using Refined 2-D Models", *Composite Structures*, 2017, doi: 10.1016/j.compstruct.2017.05.021.

[11] Fiorenzo A. Fazzolri and E. Carrera, "Thermal stability of FGM sandwich plates under various through the thickness temperature distributions", *Journal of Thermal Stresses*, vol. 37, no. 12, pp. 1449-1481, 2014.

[12] Fiorenzo A. Fazzolri, "Natural frequencies and critical temperatures of functionally graded sandwich plates subjected to uniform and non-uniform temperature distributions", *Composite Structures*, vol. 121, pp. 197–210, 2015.

[13] Yufeng Xing and Zekun Wang, "Closed Form Solutions for Thermal buckling of Functionally Graded Rectangular Thin Plates", *Applied Sciences*, vol. 7, pp. 1256-1274, 2017.

[14] Behrouz Karami, Maziar Janghorban, Davood Shahsavari and Abdelouahed Tounsi, "A size-dependent quasi-3D model for wave dispersion analysis of FG nanoplates", *Steel and Composite Structures*, vol. 28, pp. 99-110, 2018.

[15] P Emmanuel Nicholas, Dharmaraja, A Sathya Sofia and D Vasudevan, "Optimization of laminated composite plates subjected to nonuniform thermal loads", *Polymers and polymer composites*, vol. 27, no. 6, pp. 314-322, 2019.

[16] Adnan Naji Jameel, Ibtehal Abbas Sadiq and Hasanain Ibraheem Nsaif, "Buckling Analysis of Composite Plates under Thermal and Mechanical Loading", *Journal of Engineering*, vol. 18, no. 12, pp. 1365-1390, 2012.

[17] Abdelbaki Chikh, Abdelouahed Tounsi, Habib Hebali and S. R. Mahmoud, "Thermal buckling analysis of cross-ply laminated plates using a simplified HSDT", *Smart Structures and Systems,* vol. 19, no. 3, pp. 289-297, 2017.

[18] Saeed.Tahir,Abdelbaki Chikh, Abdelouahed Tounsi, Mohammed A.Al-Osta,SalahU.Al-Dulaijan, Mesfer M.Al-Zahrani, "Wave propagation analysis of a ceramic-metal functionally graded sandwich plate with different porosity distributions in a hygro-thermal environment", *Composite Structures*, vol 269, pp. 114030. 2021.

[19] Amina Attia , Amina Tahar Berrabah , Abdelmoumen Anis Bousahla , Fouad Bourada , Abdelouahed Tounsi , SR Mahmoud, "Free vibration analysis of FG plates under thermal environment via a simple 4-unknown HSDT", *Steel and Composite Structures, An*

International Journal, vol. 41, no. 6, pp. 899-910, 2021.
[20] Bouzid Merazka, Abdelhakim Bouhadra, Abderrahmane Menasria, Mahmoud M. Selim, Abdelmoumen Anis Bousahla, Fouad Bourada, Abdeldjebbar Tounsi, Kouider Halim Benrahou, Abdelouahed Tounsi and Mesfer Mohammad Al-Zahrani, "Hygro-thermo-mechanical bending response of FG plates resting on elastic foundations", *Steel and Composite Structures,* vol. 39, no. 5, pp. 631-643, 2021.
[21] Ismail Mudhaffar, Abdelouahed Tounsi, Abdelbaki Chikh, Mohammed A.Al-Osta, Salah U.Al-Dulaijan, "Hygro-thermo-mechanical bending behavior of advanced functionally graded ceramic metal plate resting on a viscoelastic foundation", *Structures*, vol. 33, pp. 2177-2189, 2021.
[22] Hichem Bellifa, Mahmoud M. Selim, Abdelbaki Chikh, Abdelmoumen Anis Bousahla, Fouad Bourada, Abdeldjebbar Tounsi, Kouider Halim Benrahou, Mesfer Mohammad Al-Zahrani and Abdelouahed Tounsi, "Influence of porosity on thermal buckling behavior of functionally graded beams", *Smart Structures and Systems*, vol. 27, no. 4, pp. 719-728, 2021.
[23] Mohamad W. Zaitoun, Abdelbaki Chikh, Abdelouahed Tounsi, Alfarabi Sharif, Mohammed A. Al-Osta, Salah U. Al-Dulaijan and Mesfer M. Al-Zahrani, "An efficient computational model for vibration behavior of a functionally graded sandwich plate in a hygrothermal environment with viscoelastic foundation effects", *Engineering with Computers,* 2021, https://doi.org/10.1007/s00366-021-01498-1.
[24] Mohammed A. Al-Osta, Hayat Saidi, Abdelouahed Tounsi, SU Al-Dulaijan, MM Al-Zahrani, Alfarabi Sharif, Abdeldjebbar Tounsi, "Influence of porosity on the hygro-thermo-mechanical bending response of an AFG ceramic-metal plates using an integral plate model", *Smart Structures and Systems, An International Journal,* vol. 28, no. 4, pp. 499-513, 2021.
[25] Ehsan Arshid, Mohammad Khorasani, Zeinab Soleimani-Javid, Saeed Amir, Abdelouahed Tounsi, "Porosity-dependent vibration analysis of FG microplates embedded by polymeric nanocomposite patches considering hygrothermal effect via an innovative plate theory", *Engineering with Computers*, 2021, doi:10.1007/s00366-021-01382-y.
[26] Yong Huang,Behrouz Karami, Davood Shahsavari &Abdelouahed Tounsi, "Static stability analysis of carbon nanotube reinforced polymeric composite doubly curved micro-shell panels", *Archives of Civil and Mechanical Engineering*, vol. 21, no. 139, pp. 1-15, 2021.
[27] Allam, Othmane ,Draiche, Kada ,Bousahla, Abdelmoumen Anis, Bourada, Fouad,Tounsi, Abdeldjebbar, Benrahou, Kouider Halim ,Mahmoud, S.R., Adda Bedia, E.A., Tounsi, Abdelouahed ," A generalized 4-unknown refined theory for bending and free vibration analysis of laminated composite and sandwich plates and shells", *Computers and Concrete*, vol. 26, no. 2, pp. 185-201, 2020.

[28] Mohamed Bourada, Abdelouahed Tounsi, Mohammed Sid Ahmed Houari and El Abbes Adda Bedia, "A new four-variable refined plate theory for thermal buckling analysis of functionally graded sandwich plates", *Journal of Sandwich Structures and Materials*, vol. 14, no. 1, pp. 5-33, 2011.

[29] Abdelkader Mahmoudi Samir Benyoucef, Abdelouahed Tounsi, Abdelkader Benachour, El Abbas Adda Bedia, S.R . Mahmoud, "A refined quasi-3D shear deformation theory for thermo-mechanical behavior of functionally graded sandwich plates on elastic foundations", *Journal of Sandwich Structures & Materials*, vol. 21, no. 6, pp. 1906-1929, 2019.

[30] Belbachir, Nasrine, Bourada, Mohamed ,Draiche, Kada,Tounsi, Abdelouahed, Bourada, Fouad, Bousahla, Abdelmoumen Anis, Mahmoud, S.R., "Thermal flexural analysis of anti-symmetric cross-ply laminated plates using a four variable refined theory", *Smart Structures and Systems,* vol. 25, no. 4, pp. 409-422, 2020.

[31] Abualnour, Moussa, Chikh, Abdelbaki, Hebali, Habib, Kaci, Abdelhakim, Tounsi, Abdeldjebbar, Bousahla, Abdelmoumen Anis, Tounsi, Abdelouahed, "Thermomechanical analysis of antisymmetric laminated reinforced composite plates using a new four variable trigonometric refined plate theory", *Computers and Concrete,* vol. 24, no. 6, pp. 489-498, 2019.

[32] Draiche, Kada, Bousahla, Abdelmoumen Anis, Tounsi, Abdelouahed, Alwabli, Afaf S.,Tounsi, Abdeldjebbar, Mahmoud, S.R., "Static analysis of laminated reinforced composite plates using a simple first-order shear deformation theory", *Computers and Concrete,* vol. 24, no. 4, pp. 369-378, 2019.

[33] Nasrine Belbachir, Kada Draich, Abdelmoumen Anis Bousahla, Mohamed Bourada, Abdelouahed Tounsi and M. Mohammadimehr, "Bending analysis of anti-symmetric cross-ply laminated plates under nonlinear thermal and mechanical loadings", *Steel and Composite Structures*, vol. 33, no.1, pp. 81-92, 2019.

[34] Meriem Sahla, Hayat Saidi, Kada Draiche, Abdelmoumen Anis Bousahla, Fouad Bourada and Abdelouahed Tounsi, "Free vibration analysis of angle-ply laminated composite and softcore sandwich plates", *Steel and Composite Structures*, vol. 33, no. 5, pp. 663-679, 2019.

[35] J.L. Mantari, A.S. Oktem and C. Guedes Soares, "Static and dynamic analysis of laminated composite and sandwich plates and shells by using a new higher-order shear deformation theory", *Composite Structures*, vol. 94, pp. 37–49, 2011.

[36] Widad Ibraheem Majeed and Ibtehal Abbas Sadiq, "Buckling and Pre Stressed Vibration Analysis of Laminated Plates Using New Shear Deformation", *IOP Conf. Series: Materials Science and Engineering*, Istanbul, August, vol. 454, pp. 012006, 2018.

[37] Widad Ibraheem Majeed, "Thermal buckling analysis of cross-ply plates based on new displacement field", *Journal of Engineering Research*, vol. 9, no. 3B, pp. 302-316, 2021.
[38] J.N. Reddy, Mechanics of Laminated Composite Plates and Shells, 2nd ed, CRC press, USA, 2004.
[39] Kari R. Thangaratnam, Palaninathan and J. Ramachandran, "Thermal Buckling of Composite Laminated plates", *Computers and Structures*, vol. 32, no. 5, pp. 1117–1124, 1989.
[40] Ibtehal Abbas Sadiq and Widad Ibraheem Majeed, "Thermal Buckling of Angle-Ply Laminated Plates Using New Displacement Function", *Journal of Engineering*, vol. 25, no. 12, pp. 96-113, 2019.